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TMD Universality and Evolution

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Based on work with Collins, Ji, Kang, Kouvaris, Sterman, Vogelsang, Yuan, …

Outline

- ❑ Predictive power of QCD calculation
- ❑ Factorization vs non-factorization
- ❑ Necessary condition for pQCD factorization
- ❑ TMD and Collinear factorization

  Collinear factorization + Sudakov resummation

  =\= TMD factorization
- ❑ Universality of TMD/Collinear PDFs
- ❑ Evolution of TMD/Collinear PDFs and correlations
- ❑ Summary and outlook

Foundation of perturbative QCD

- Renormalization
  – QCD is renormalizable

Nobel Prize, 1999
't Hooft, Veltman

- Asymptotic freedom
  – weaker interaction at a shorter distance

Nobel Prize, 2004
Gross, Politzer, Wilczek

- Infrared safety
  – pQCD factorization and calculable short distance dynamics

J.J. Sakurai Prize
Mueller, Sterman, 2003
Collins, Ellis, Soper, 2009

Predictive power of QCD calculation

❑ Necessary conditions to apply pQCD:

- ✧ Observable has a large momentum transfer: $Q \gg m, \Lambda_{\rm QCD} \sim 1/{\rm fm}$
- ✧ Observable is IR safe when all hadronic mass scales go to zero

❑ Cross section with NO identified hadron:

- ✧ Hadronic total cross section in e^+e^- collisions

$$\sigma_{e^+e^-\to{\rm hadrons}}(Q=\sqrt{S}) = \sigma_{e^+e^-\to{\rm partons}}(Q)$$

- ✧ Inclusive jet cross section in e^+e^- collisions

$$\sigma_{e^+e^-\to{\rm jets}}(Q=\sqrt{S}, R_{\rm cone}, E_h)$$

❑ Cross section with one or more identified hadron:

- ✧ PQCD does not work for the dynamics at hadronic scale: 1/fm
- ✧ PQCD alone cannot predict such a cross section

$\Longrightarrow$ **Factorization?!**

Factorization vs non-factorization

❑ Cross section with ONE identified hadron:

$$\sigma_h(Q,M,S_h)=\sum_n \sigma_h^n(S_h)\left(\frac{M}{Q}\right)^n \approx \sum_n C_n(Q/\mu,S_h)\,\underbrace{\langle h,S_h|\mathcal{O}_n|h,S_h\rangle}_{\text{Function of } \mu,\, M}\left(\frac{M}{Q}\right)^n$$

Factorization to all powers

OPE

❑ Cross section with TWO or more identified hadron:

- ✧ OPE does not help for factorization – $\langle h_1,h_2|\mathcal{O}_n(0)|h_1,h_2\rangle$
- ✧ Inclusive Drell-Yan cross section is not fully factorizable!

$$\sigma^{\rm DY}_{h_1h_2}(Q,M)\approx \boxed{C^{(0)}_{ab}(Q/\mu,\alpha_s(\mu))\otimes f^{(2)}_{a/h_1}\otimes f^{(2)}_{b/h_2}}$$
$$+\frac{1}{Q^2}C^{(2)}_{ab}(Q/\mu,\alpha_s(\mu))\otimes f^{(2)}_{a/h_1}\otimes f^{(4)}_{b/h_2}+(h_1\leftrightarrow h_2)+\frac{1}{Q^4}\mathcal{C}^{(4)}_{h_1h_2}+\dots$$

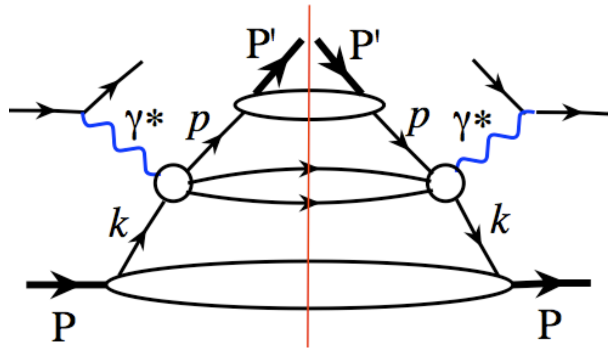
❑ Predictive power of pQCD factorization:

- ✧ Universality of PDFs
- ✧ Evolution of PDFs

“Right” to neglect non-factorizable power corrections

The necessary condition for factorization

- Experiments do not see partons directly:



$$\propto \int d^4k \int d^4p$$

- Perturbative pinch singularity:

$$I_k = \int d^4k \; \mathcal{J}_h(P,k) \, \frac{1}{k^2 + i\epsilon} \, \frac{1}{k^2 - i\epsilon} \, \mathcal{H}(k,Q,p)$$

$$\approx \int d^4k \; \mathcal{J}_h(P,k) \, \frac{1}{k^2 + i\epsilon} \, \frac{1}{k^2 - i\epsilon} \, \left[\mathcal{H}(k^2=0,Q,p) + \ldots\right]$$

Dominated by $k^2 \sim 0$

Similar approximation for the p-integration

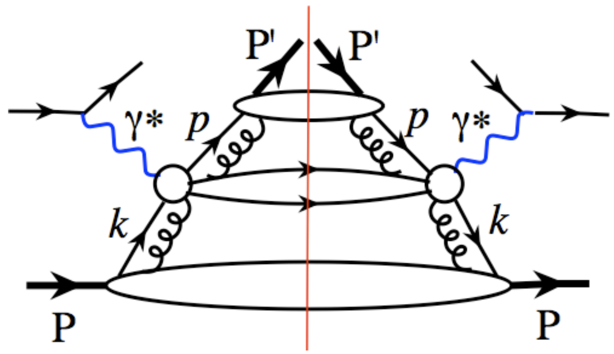
Cross section is factorized into 3 parts: $J_h(P), J_{h'}(P'), H(k^2=0,Q,p^2=0)$

They are separated by long-lived parton states comparing to 1/Q

Long-lived parton state – necessary condition for Factorization

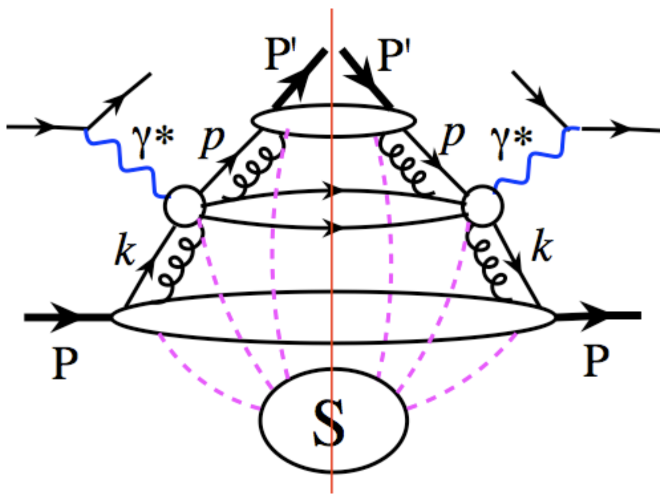
Other leading contributions

❑ Collinear gluons:



Collinear longitudinally polarized gluons do not change the collinear collision kinematics

❑ Soft interaction:



If the interaction between two jet functions can resolve the "details" of the jet functions, the jet functions could be altered before hard collision – break of the factorization

TMD and collinear factorization

- On-shell parton entering the hard part:

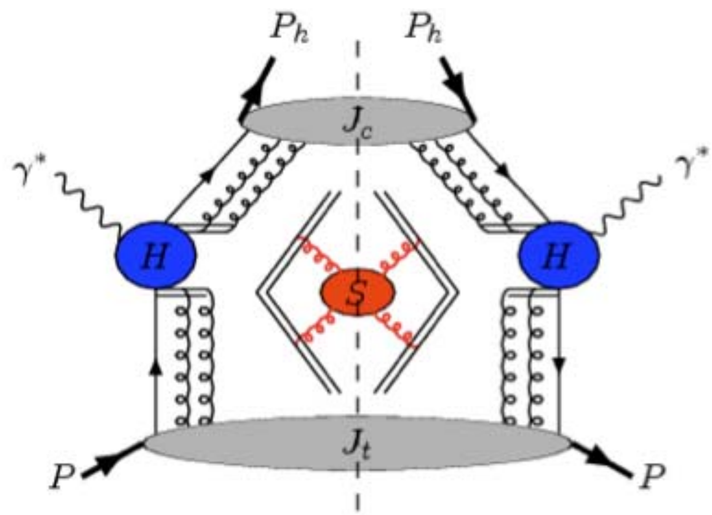
$$d^4k \Rightarrow \frac{dk^+}{k^+} d^2k_T\ dk^2 = \frac{dx}{x} d^2k_T\ dk^2$$

Included in TMD PDFs

- TMD factorization:

$$\sigma_{\rm SIDIS}(Q, P_{h\perp}) = \hat{H}(Q) \otimes \Phi_f \otimes \mathcal{D}_{f\to h} \otimes \mathcal{S} + \mathcal{O}\left(\frac{P_{h\perp}}{Q}\right)$$

$$\vec{P}_{h\perp} = \vec{k}_{i\perp} + \vec{k}_{f\perp} + \vec{k}_{s\perp}$$



- Collinear factorization:

IF $Q \gg k_\perp, \sqrt{k^2}$,

$$d^4k \Rightarrow \frac{dk^+}{k^+} d^2k_T\ dk^2 = \frac{dx}{x} d^2k_T\ dk^2$$

Included in PDFs

$$\mathcal{S} = 1$$

TMD vs collinear factorization

- "Formal" operator relation between TMD distributions and collinear factorized distributions:

spin-averaged: $$\int d^2k_\perp \Phi_f^{\rm SIDIS}(x,k_\perp) + {\rm UVCT}(\mu_F^2) = \phi_f(x,\mu_F^2)$$

Transverse-spin: $$\frac{1}{M_P}\int d^2k_\perp\, \vec{k}_\perp^2\, q_T(x,k_\perp) + {\rm UVCT}(\mu_F^2) = T_F(x,x,\mu_F^2)$$

But, TMD factorization is only valid for low k_T– TMD PDFs at low k_T

- TMD factorization and collinear factorization cover different regions of kinematics:

Collinear: $Q_1 \ldots Q_n >> \Lambda_{QCD}$

TMD: $Q_1 >> Q_2 \sim \Lambda_{QCD}$

✧ One complements the other, but, cannot replace the other!

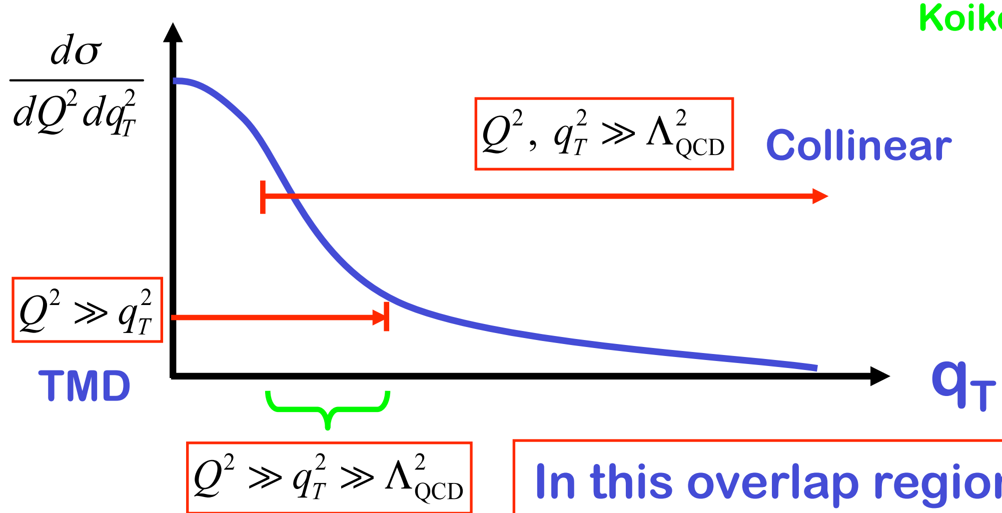
✧ Predictive power of both formalisms relies on the validity of their own factorization

Consistency check – overlap region – perturbative region

The consistency check

- **IF** both factorizations are proved to be valid,
  - ✧ both formalisms should yield the same result in overlap region
  - ✧ Case studies – Drell-Yan/SIDIS

Ji, Qiu, Vogelsang, and Yuan
Koike, Vogelsang, and Yuan



In this overlap region, both formalisms indeed give the same result

- **IF** one factorization formalism is valid,

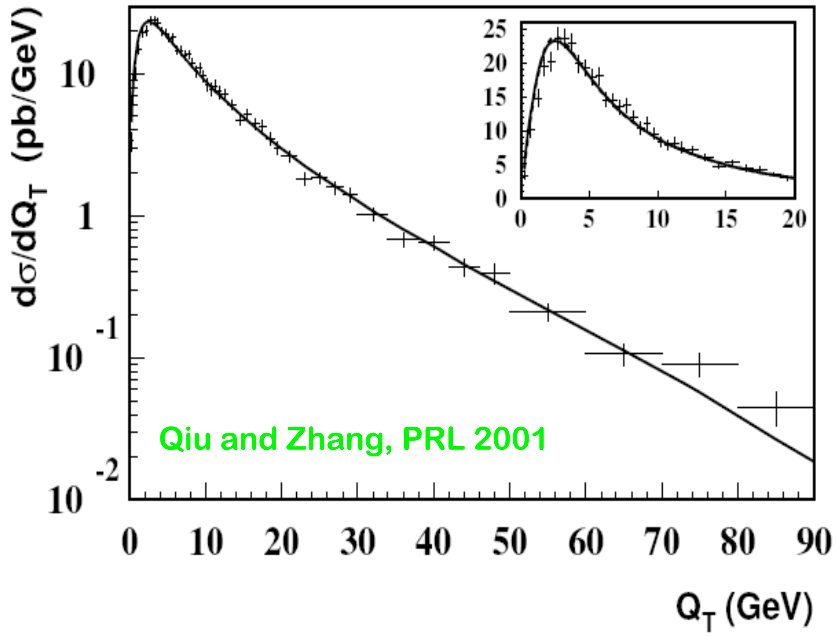
Qiu, Vogelsang, and Yuan

  - ✧ Its asymptotic form in the overlap region is a necessary condition for the other formalism to match
  - ✧ But, it is not sufficient to prove the other factorization formalism

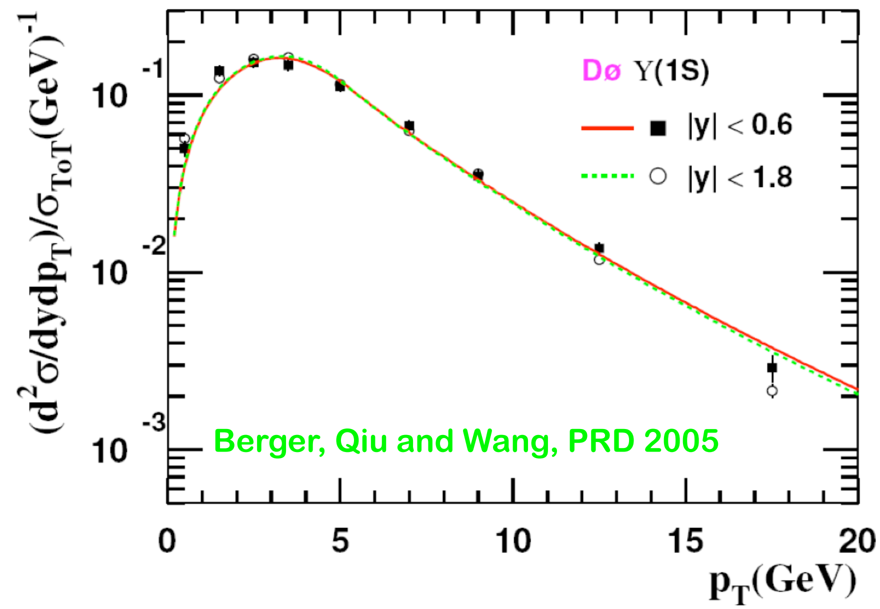
Question

Can collinear factorization + resummation mimic/replace TMD factorization?

- Fermilab CDF data on Z at $\sqrt{S} = 1.8$ TeV



- Fermilab D0 Run II data on Upsilon at √S=1.96 TeV



Answer:

Collinear factorization + Sudakov resummation =\= TMD factorization

Collinear factorization + Resummation

- **Resummation of large logarithms is a reorganization of the perturbative series in collinear factorization**
  - ✧ It improves the predictive power of collinear factorization to a wider kinematic regime
  - ✧ It does not include ANY O(k_T)-TMD distributions at leading twist e.g.

$$\langle P,S|\bar{\psi}\sigma^{+\perp}\psi|P,S\rangle + \langle P,-S|\bar{\psi}\sigma^{+\perp}\psi|P,-S\rangle = 0$$

if two fields on the same light-cone

⟶ NO Boer-Mulders function in LT collinear factorization

$$\langle P,S|\bar{\psi}\gamma^{+}\psi|P,S\rangle - \langle P,-S|\bar{\psi}\gamma^{+}\psi|P,-S\rangle = 0$$

⟶ NO Sivers function in LT collinear factorization

- **TMD distribution at low k_T includes rich information of HT contributions of collinear factorization**

JLab12, …, and EIC can explore the rich transition region

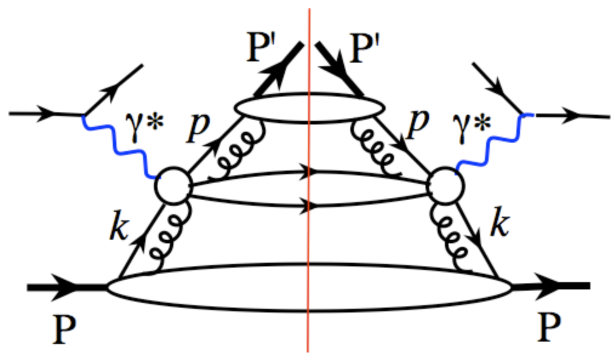
Universality of PDFs

❑ Collinear factorization:

All leading twist PDFs and high twist multiparton correlations are independent of details of partonic processes – universal

❑ TMD factorization:

TMD distributions could be process dependent due to the leading power contribution of collinear gluons – gauge link



❑ Gauge link is sensitive to the path in gauge field theory:

Sum of gauge links along a close path is not equal to the unity

TMD parton distributions

- **SIDIS:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2\mathbf{y}_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp}\, \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\Phi_n^\dagger(\{\infty,0\},\mathbf{0}_\perp)$$
$$\times\, \Phi_{\mathbf{n}_\perp}^\dagger(\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\frac{\gamma^+}{2}\Phi_n(\{\infty,y^-\},\mathbf{y}_\perp)\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

Gauge links:

$$\Phi_n(\{\infty,y^-\},\mathbf{y}_\perp) \equiv \mathcal{P}e^{-ig\int_{y^-}^{\infty} dy_1^- n^\mu A_\mu(y_1^-,\mathbf{y}_\perp)}$$
$$\Phi_{\mathbf{n}_\perp}(\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\}) \equiv \mathcal{P}e^{-ig\int_{\mathbf{0}_\perp}^{\mathbf{y}_\perp} d\mathbf{y}'_\perp \mathbf{n}_\perp^\mu A_\mu(\infty,\mathbf{y}'_\perp)}$$

- **Drell-Yan:**

$$f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2\mathbf{y}_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp}\langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\Phi_n^\dagger(\{-\infty,0\},\mathbf{0}_\perp)$$
$$\times\, \Phi_{\mathbf{n}_\perp}^\dagger(-\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\frac{\gamma^+}{2}\,\Phi_n(\{-\infty,y^-\},\mathbf{y}_\perp)\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

- **PT invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\beta|\alpha\rangle \qquad f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

Collins 2002
Boer et al, 2003
Kang, Qiu, 2009

- **Sivers function:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)\,\vec{S}\cdot(\hat{p}\times\hat{\mathbf{k}}_\perp)$$

$$\Longrightarrow \quad f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)^{\rm SIDIS} = -f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)^{\rm DY} \quad \Longleftarrow$$

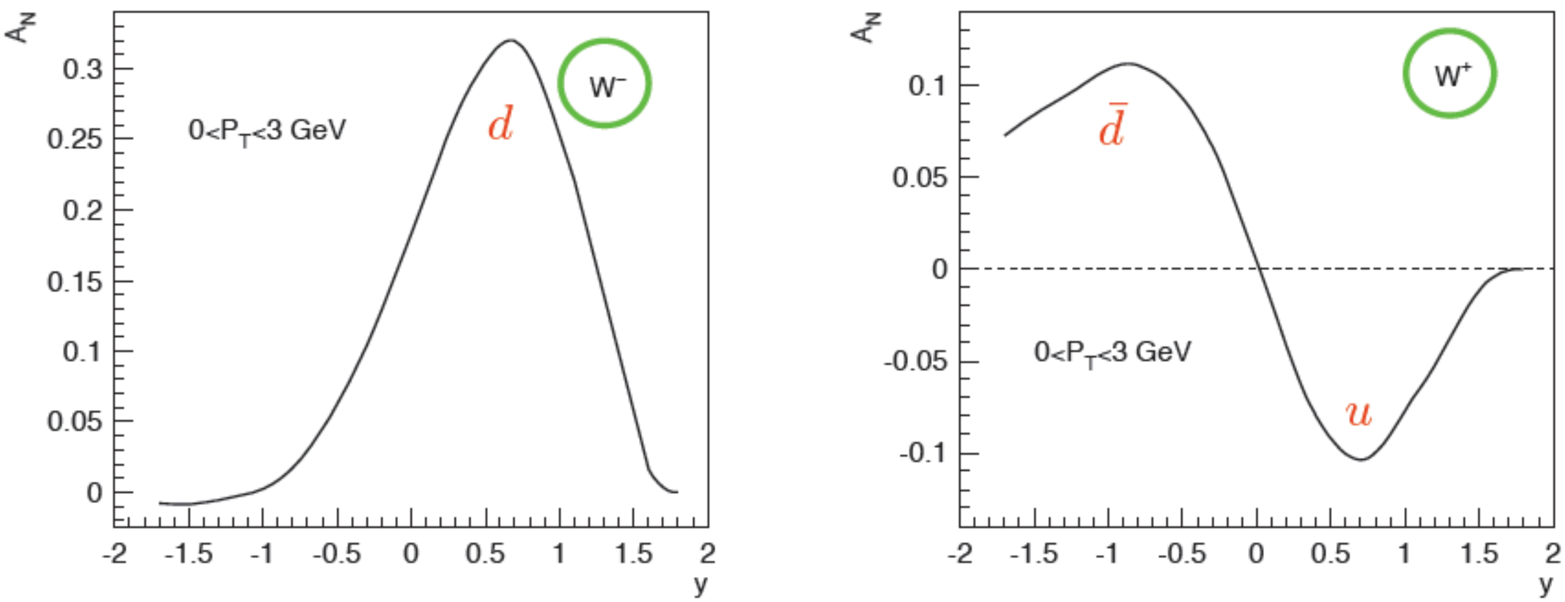
Modified Universality

Test of the modified universality

- **SSA of W-production at RHIC :**

Kang, Qiu, 2009

Sivers function same as DY, different from SIDIS by a sign



- flavor separation
- large asymmetry: should be able to see sign change

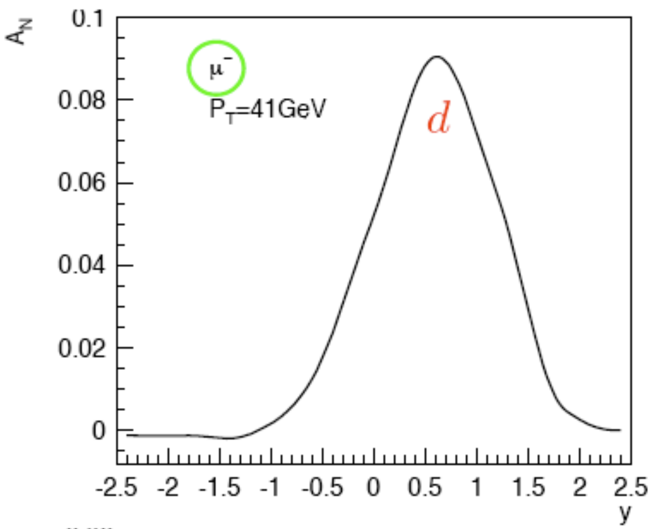
But, the detectors at RHIC cannot reconstruct the W's

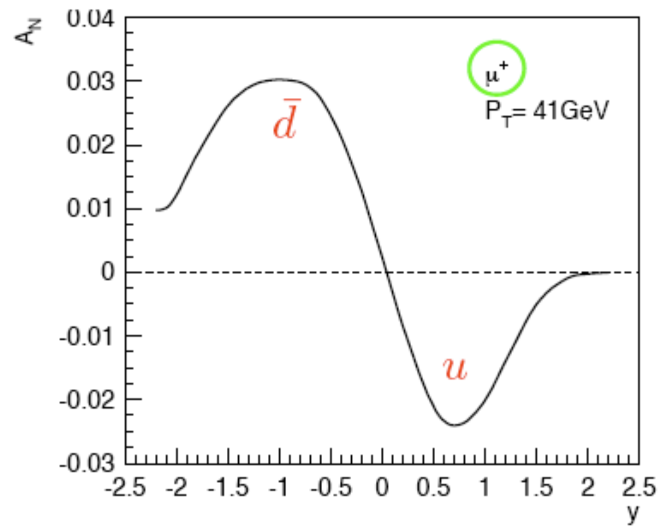
The Sivers functions from Anselmino et al 2009

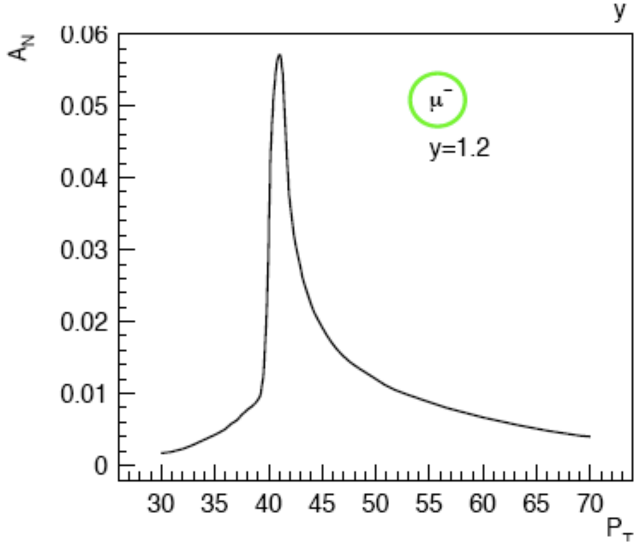
SSA of lepton from W-decay

Kang, Qiu, 2009

❑ Lepton SSA is diluted from the decay:







- flavor separation
- asymmetry gets smaller due to dilution
  should still be measurable by current
  RHIC sensitivity

Larger SSA for Z^0 production while the rate is lower

Evolution of PDFs

❑ Factorization scale dependence:

$$\sigma_{\text{Physical}}(Q) \approx \mathcal{H}(Q/\mu_F, Q/\mu, \alpha_s(\mu)) \otimes \phi_f(\mu_F, \mu) \otimes \ldots$$

✧ μ_F-dependence controls how much "collinear" partonic radiative correction should be included in the "hard" part

✧ μ_F-dependence of PDFs should cancel the μ_F-dependence of H

❑ Evolution of TMD distributions:

✧ NO evolution equation has been derived for TMD distributions in x and k_T space

✧ Evolution equation was derived for Fourier transformed TMD distributions when b small

$$\widetilde{\Phi}(x, b_\perp, \mu, x\zeta) = \int d^2k_\perp \, e^{i\vec{k}_\perp \cdot \vec{b}_\perp} \, \Phi(x, k_\perp, \mu, x\zeta)$$

$$\zeta \frac{\partial}{\partial \zeta} \widetilde{\Phi}(x, b_\perp, \mu, x\zeta) \sim (K(\mu, b_\perp) + G(\mu, x\zeta)) \, \widetilde{\Phi}(x, b_\perp, \mu, x\zeta) \qquad \zeta = \frac{(2P \cdot n)^2}{n^2}$$

$$\mu \frac{d}{d\mu} K = -\gamma_K = -\mu \frac{d}{d\mu} G$$

⟹ Double log resummation in b-space

Collins and Soper, 1981

Collinear factorization for one scale SSA

- QCD Collinear factorization approach is more relevant

$$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$$ – Expansion

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

Too large to compete!

Three-parton correlation

- SSA – difference of two cross sections with spin flip is power suppressed compared to the cross section

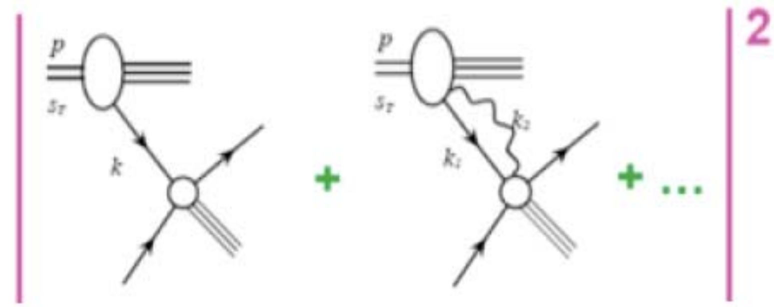
$$\Delta\sigma(Q, s_T) \equiv [\sigma(Q, s_T) - \sigma(Q, -s_T)]/2$$
$$= (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) + \mathcal{O}(1/Q^2)$$

- Sensitive to twist-3 multi-parton correlation functions
- Integrated information on parton's transverse motion

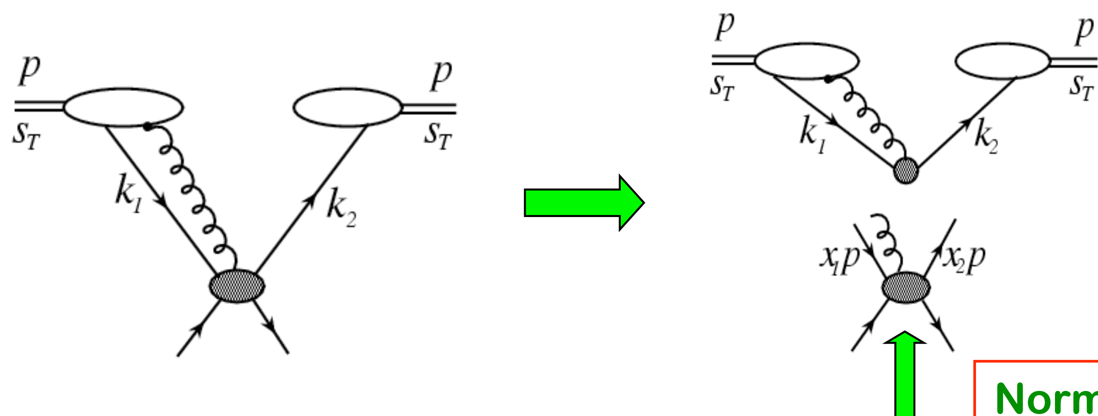
SSA in QCD Collinear Factorization – I

- **All scales >> Λ_{QCD}:**

$$\sigma(s_T) \sim$$



- **Factorization at twist-3 – initial-state:**



Normal twist-2 distributions

- **Twist-3 quark-gluon correlation:**

$$T_{q,F}(x,x,\mu_F) = \int \frac{dy_1^-}{2\pi} e^{ixP^+y_1^-} \langle P, s_T | \overline{\psi}_q(0) \frac{\gamma^+}{2} \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} F_\sigma^{\ +}(y_2^-) \right] \psi_q(y_1^-) | P, s_T \rangle$$

Spin flip: $g_1 \leftrightarrow \gamma^+\gamma^5,$ $\quad h_1 \leftrightarrow \sigma^{+\perp},$ $\quad T_F \leftrightarrow \gamma^+(?)$

SSA in QCD Collinear Factorization – II

Qiu, Sterman, 1998

❑ Factorization formalism for SSA of single hadron:

$$
\begin{aligned}
\Delta\sigma_{A+B\to\pi}(\vec{s}_T) = &\sum_{abc} \phi^{(3)}_{a/A}(x_1, x_2, \vec{s}_T) \otimes \phi_{b/B}(x') \otimes H_{a+b\to c}(\vec{s}_T) \otimes D_{c\to\pi}(z) \\
&+ \sum_{abc} \delta q^{(2)}_{a/A}(x, \vec{s}_T) \otimes \phi^{(3)}_{b/B}(x'_1, x'_2) \otimes H''_{a+b\to c}(\vec{s}_T) \otimes D_{c\to\pi}(z) \\
&+ \sum_{abc} \delta q^{(2)}_{a/A}(x, \vec{s}_T) \otimes \phi_{b/B}(x') \otimes H'_{a+b\to c}(\vec{s}_T) \otimes D^{(3)}_{c\to\pi}(z_1, z_2) \\
&+ \text{higher power corrections},
\end{aligned}
$$

Only one twist-3 distribution in each term!

- ❖ 1st term: Collinear version of Sivers effect
- ❖ 2nd term: Collinear version of transversity + BM function
- ❖ 3rd term: Collinear version of Collins effect

Scale dependence of SSA

- **Almost all existing calculations of SSA are at LO:**
  - Strong dependence on renormalization and factorization scales
  - Artifact of the lowest order calculation
- **Improve QCD predictions:**
  - Complete set of twist-3 correlation functions relevant to SSA
  - LO evolution for the universal twist-3 correlation functions
  - NLO partonic hard parts for various observables
  - NLO evolution for the correlation functions, …
- **Current status:**
  - Two sets of twist-3 correlation functions
  - LO evolution kernel for $T_{q,F}(x,x)$ and $T_{G,F}^{(f,d)}(x,x)$ Kang, Qiu, 2009; Braun et al, 2009
  - NLO hard part for SSA of p_T weighted Drell-Yan Vogelsang, Yuan, 2009

Two sets of twist-3 correlation functions

❑ Twist-2 distributions:

❖ **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

❖ **Polarized PDFs:**

$$\Delta q(x) \propto \langle P,S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P,S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P,S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:

Kang, Qiu, PRD, 2009

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}^{(f,d)}_{G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}^{(f,d)}_{\Delta G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Evolution equations and evolution kernels

- Evolution equation is a consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

- Evolution kernel is process independent:
  - Calculate directly from the variation of process independent twist-3 distributions

    Kang, Qiu, 2009
    Yuan, Zhou, 2009
  - Extract from the scale dependence of the NLO hard part of any physical process

    Vogelsang, Yuan, 2009
  - Both approaches should give the same kernel
  - UV Renormalization of the twist-3 operators

    Braun et al, 2009

Leading order evolution equations – I

Kang, Qiu, PRD, 2009

❑ Quark:

$$\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{\boxed{P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu_F)}$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F)-T_{q,F}(\xi,\xi,\mu_F)\right]+z\,T_{q,F}(\xi,x,\mu_F)\right]+\frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right]$$
$$+P_{qg}(z)\left(\frac{1}{2}\right)\boxed{\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)+T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]}\Bigg\}$$

❑ Antiquark:

$$\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{\boxed{P_{qq}(z)\,T_{\bar{q},F}(\xi,\xi,\mu_F)}$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F)-T_{\bar{q},F}(\xi,\xi,\mu_F)\right]+z\,T_{\bar{q},F}(\xi,x,\mu_F)\right]+\frac{C_A}{2}\left[T_{\Delta\bar{q},F}(x,\xi,\mu_F)\right]$$
$$+P_{qg}(z)\left(\frac{1}{2}\right)\boxed{\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)-T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]}\Bigg\}$$

- ❖ All kernels are infrared safe
- ❖ Diagonal contribution is the same as that of DGLAP
- ❖ Quark and antiquark evolve differently – caused by tri-gluon

Leading order evolution equations – II

Kang, Qiu, PRD, 2009

❑ Gluons:

$$\frac{\partial T^{(d)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{\boxed{P_{gg}(z)\,T^{(d)}_{G,F}(\xi,\xi,\mu_F)}\right.$$
$$+\frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(d)}_{G,F}(\xi,x,\mu_F)-T^{(d)}_{G,F}(\xi,\xi,\mu_F)\right]\right.$$
$$\left.+2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(d)}_{G,F}(\xi,x,\mu_F)+(1+z)\,T^{(d)}_{\Delta G,F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{gq}(z)\left(\frac{N_c^2-4}{N_c^2-1}\right)\boxed{\sum_q\left[T_{q,F}(\xi,\xi,\mu_F)+T_{\bar{q},F}(\xi,\xi,\mu_F)\right]}\right\}$$

Similar expression for $T^{(f)}_{G,F}(x,x,\mu_F)$

- ❖ Kernels are also infrared safe
- ❖ diagonal contribution is the same as that of DGLAP
- ❖ Two tri-gluon distributions evolve slightly different
- ❖ $T^{(d)}_{G,F}$ has no connection to TMD distribution
- ❖ Evolution can generate $T^{(d)}_{G,F}$ as long as $\sum_q \left[T_{q,F}+T_{\bar{q},F}\right]\neq 0$

Leading order evolution equations – III

- Evolution equations for diagonal correlation functions are not closed!
- Model for the off-diagonal correlation functions:

For the symmetric correlation functions:

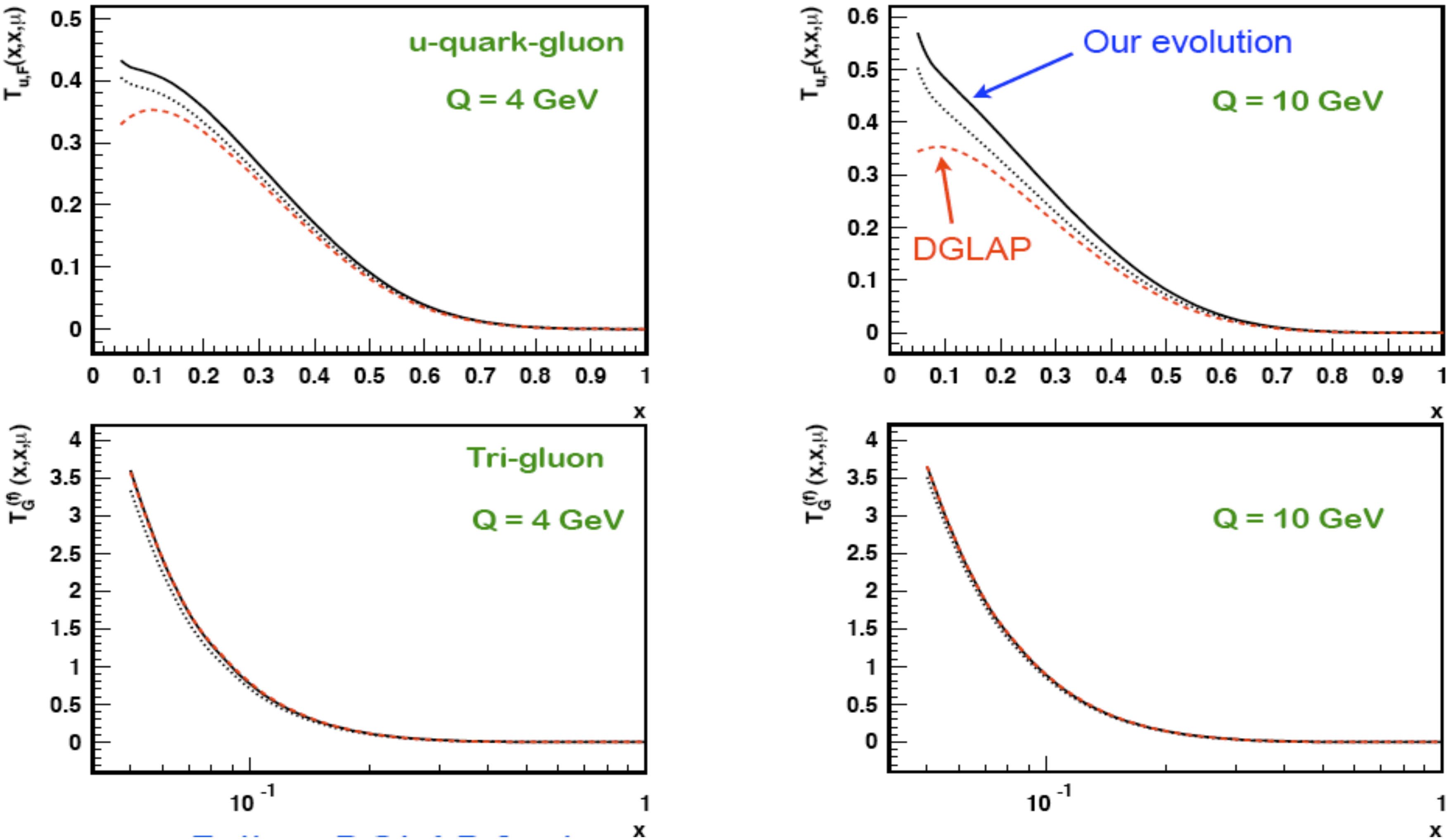
$$T_{q,F}(x_1, x_2, \mu_F) = \tfrac{1}{2}[T_{q,F}(x_1, x_1, \mu_F) + T_{q,F}(x_2, x_2, \mu_F)]e^{-[(x_1-x_2)^2/2\sigma^2]},$$

$$\mathcal{T}^{(f,d)}_{G,F}(x_1, x_2, \mu_F) = \tfrac{1}{2}[\mathcal{T}^{(f,d)}_{G,F}(x_1, x_1, \mu_F) + \mathcal{T}^{(f,d)}_{G,F}(x_2, x_2, \mu_F)]e^{-[(x_1-x_2)^2/2\sigma^2]},$$

$\Longrightarrow$

$$T^{(f,d)}_{G,F}(x_1, x_2, \mu_F) = \frac{1}{2}\left[T^{(f,d)}_{G,F}(x_1, x_1, \mu_F) + \frac{x_2}{x_1}T^{(f,d)}_{G,F}(x_2, x_2, \mu_F)\right]e^{-[(x_1-x_2)^2/2\sigma^2]}.$$

Scale dependence of twist-3 correlations



- ❖ Follow DGLAP at large x
- ❖ Large deviation at low x (stronger correlation)

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Summary and outlook

- Collinear factorization and TMD factorization cover different regions of kinematics

  Two are complementary to each other

  One cannot replace the other!

- TMD factorization leads to more TMD Two-parton correlations. These "New" TMD correlations connect to HT quark-gluon correlations in collinear factorization

  No evolution equation for TMD distribution in x and k_T space

- Exploring the transition region between the TMD and Collinear factorization should be very interesting

  – JLab12, …, and EIC can definitely help!

Thank you!

Backup transparencies

Factorization – beyond leading power – I

- **Cross section has the same dimension:**

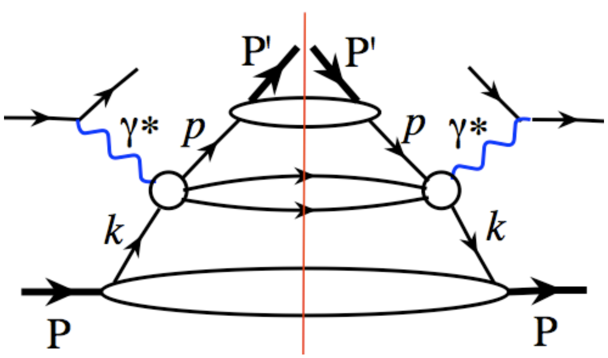
$$\sigma^{\rm DY}_{h_1h_2}(Q,M) \approx C^{(0)}_{ab}(Q/\mu,\alpha_s(\mu)) \otimes f^{(2)}_{a/h_1} \otimes f^{(2)}_{b/h_2} + \frac{1}{Q^2} C^{(2)}_{ab}(Q/\mu,\alpha_s(\mu)) \otimes f^{(2)}_{a/h_1} \otimes f^{(4)}_{b/h_2} + (h_1 \leftrightarrow h_2) + \frac{1}{Q^4}\mathcal{C}^{(4)}_{h_1h_2} + \ldots$$

Dimension of the power suppression is matched by the dimension of high twist matrix elements – multi-parton correlation functions

- **In collinear factorization, hadron mass does not enter the power expansion of partonic scattering:**

$$P^2 = M^2 \approx 0 \Rightarrow k^2 = 0, p^2 = 0, \ldots$$

- **One active parton subprocess contributes to all power**



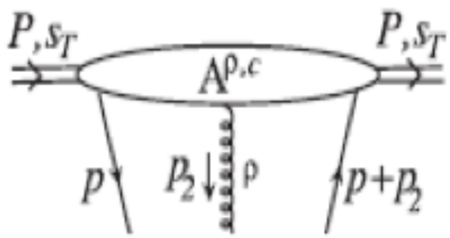
$$\mathcal{H}(k,Q,p) \approx \mathcal{H}(k^2=0,Q,p^2=0) + \mathcal{O}\left(\frac{\langle k^2\rangle, \langle p^2\rangle}{Q^2}\right)$$

$$\approx \mathcal{H}(k=xP,Q,p=P'/z) + \mathcal{O}\left(\frac{\langle k_T^2\rangle, \langle p_T^2\rangle}{Q^2}\right)$$

$$\Longrightarrow \langle P|\bar{\psi}\Gamma\partial^\mu \ldots \partial^\nu\psi|P\rangle \qquad + \mathcal{O}\left(\frac{\langle k^2\rangle, \langle p^2\rangle}{Q^2}\right)$$

Factorization – beyond leading power – II

- **Multiparton correlation functions – more active partons:**



- ✧ Gluon field operator is not a good operator
- ✧ Need contribution from two parton process
- ✧ building blocks: $\psi,\ \bar{\psi},\ D_T,\ F^{+T}$

- **Kinematics fix only one active momentum fraction:**

  Let $p \approx xP,\ p_2 \approx x_2 P,$ either x or x_2 can be very soft

- **Key difficulty in factorization beyond leading power:**

  Impossible to separate the "soft" gluon from the "zero" momentum active parton in multiparton correlation functions

- **"Only" collinearly factorizable power correction:**

  Power correction with "only" one non-leading distribution!

  Qiu and Sterman, 1991

  – Factorize the soft interaction from all leading PDFs

Evolution equations – I

- Feynman diagram representation of twist-3 distributions:

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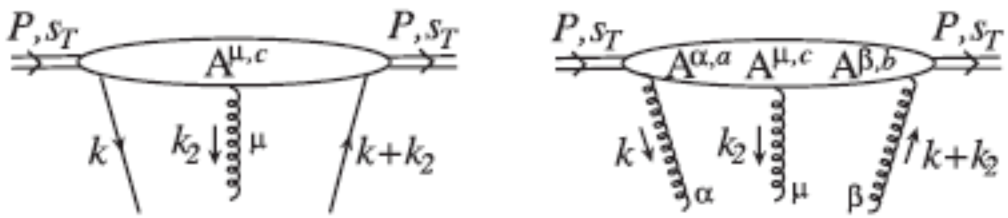
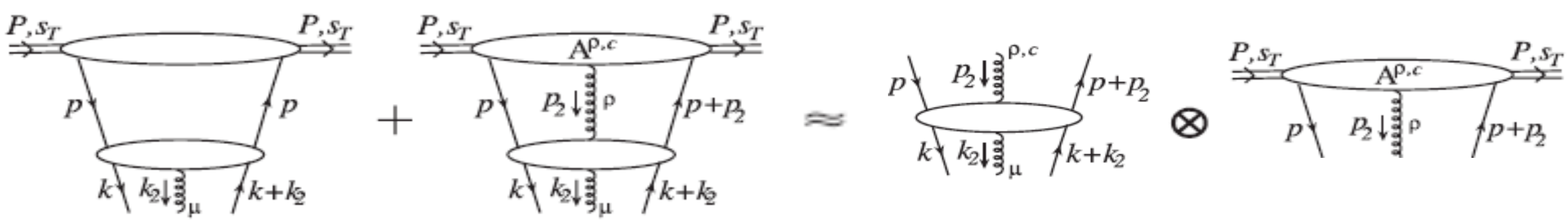


Different twist-3 distributions ⇔ diagrams with different cut vertices

- Collinear factorization of twist-3 distributions:



- Cut vertex and projection operator in LC gauge:

$$\mathcal{V}^{\,\mathrm{LC}}_{q,F} = \frac{\gamma^+}{2P^+}\,\delta\!\left(x-\frac{k^+}{P^+}\right)x_2\,\delta\!\left(x_2-\frac{k_2^+}{P^+}\right)(i\epsilon^{s_T\sigma n\bar{n}})[-g_{\sigma\mu}]\mathcal{C}_q$$

$$\mathcal{P}^{\,(\mathrm{LC})}_{q,F} = \frac{1}{2}\gamma\cdot P\left(\frac{-1}{\xi_2}\right)(i\epsilon^{s_T\rho n\bar{n}})\tilde{\mathcal{C}}_q$$

Evolution equations – II

❑ Closed set of evolution equations (spin-dependent):

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) = \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{qq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{q\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$
$$+ \sum_{i=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(i)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{qg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{q\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)].$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) = \sum_{j=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(j)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{gg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}^{(j)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{g\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$
$$+ \sum_q \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{gq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{g\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)],$$

Plus two more equations for:

$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T)$ and $\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T)$

Evolution equations – III

- Distributions relevant to SSA:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}_{q,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) + \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}^{(i)}_{G,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) + \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}_{\Delta q,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T) - \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T) - \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x+x_2, x, \mu_F, s_T)\right].$$

- Important symmetry property:

$$T_{\Delta q,F}(x, x, \mu_F) \equiv \int dx_2 [2\pi\delta(x_2)] \mathcal{T}_{\Delta q,F}(x, x+x_2, \mu_F) = 0,$$

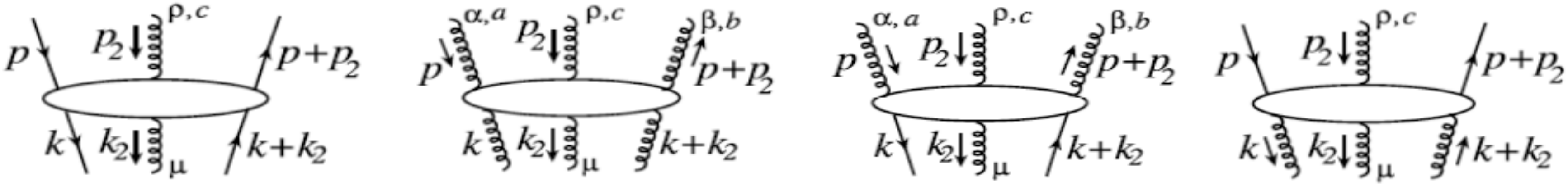
$$T^{(f,d)}_{\Delta G,F}(x, x, \mu_F) \equiv \int dx_2 [2\pi\delta(x_2)] \left(\frac{1}{x}\right) \mathcal{T}^{(f,d)}_{\Delta G}(x, x+x_2, \mu_F) = 0.$$

These two correlation functions do not give the gluonic pole contribution directly

Evolution kernels

Kang, Qiu, PRD, 2009

- Feynman diagrams:



- LO for flavor non-singlet channel: